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DETERMINATION OF ORIENTATION AND LOCATION OF PLANE DEFECTS - A TRIGONOMETRICAL APPROACH

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DETERMINATION OF ORIENTATION AND LOCATION OF PLANE DEFECTS - A TRIGONOMETRICAL APPROACH

by

D.K. Mak

ABSTRACT

The tandem method and the double transducer oblique beam method are formulated to locate defects oriented in any direction. They can be applied to defect mapping in automatic ultrasonic testing.

DÊTERMINATION DE L'ORIENTATION ET DE LA LOCATION DU PLAN DES DÉFAUTS PAR APPROCHE TRIGONOMÊTRIQUE

par

D.K. Mak

RÉSUMÉ

La méthode par transmission et celle du transducteur double à faisceau oblique sont formulées pour localiser des défauts orientés dans n'importe quel direction. Elles peuvent être appliquées pour faire une carte de défauts lors de tests automatisés par ultrason.

INTRODUCTION

Defects in material can be ultrasonically detected by (a) the normal beam method (b) single transducer oblique beam method (c) double transducer oblique beam method and (d) the tandem method. By measuring the travel time, t, between the transmitted pulse and the echo reflected, scattered or diffracted from the defect, the location of the defect can be determined.

The first two methods employ one transducer while the last two methods employ two transducers.

Methods (a), (b) and (d) were used by Udagawa et al. [1] to locate weld defects in automatic ultrasonic testing. Method (a) can be considered as a particular case of method (b) when the angle of incidence of sound beam is 90°. Method (b) was discussed in detail by Seiger [2] who attempted to locate flaws more accurately. Method (d), the tandem method has been used so far for detecting planar defects normal to the surface of the material (Fig. 1). But the method can be generalized to detect inclined defects. This generalization will be described below. We also show that, for certain orientations of the defects, the tandem method cannot be used and may be complemented by method (c), the double transducer oblique beam method.

Let (x, z) be the location of a defect inside a piece of material of thickness d. The defect is oriented at an angle α with respect to the positive x-axis. The bottom surface of the material coincides with $z = 0$. The positions of the transmitting and receiving tranducers are known. Time t can be measured. For contact measurement, the locations of the transmitter and receiver are denoted by (p_1, d) and (p_2, d). The distance traversed between the entry point and the exit point of the sound beam in the material is ℓ. The probe angle is θ. For immersion measurement, locations of the transmitter and receiver are denoted by (r_1, s_1) and (r_2, s_2).

The distance traversed between the transmitter and the entry point is ℓ_1. The distance traversed between the exit point and the receiver is ℓ_2. The angle of incidence of the sound beam from the transmitter is θ_1.

In order to formulate the equations such that they are independent of the receiver being located on the left or the right of the transmitter, the angles have to be described in standard position in a Cartesian coordinate system, i.e., with the vertex of the angle at the origin, the initial side on

the + x-axis and the terminal side wherever it may fall in one of the four quadrants. Positive angles are measured counter clockwise; negative angle, clockwise. Thus, a transducer with probe angle θ pointing downward to the left i.e., in the third quadrant, will be described by $\theta' = 3\pi/2 - \theta$. If it is pointing downward to the right, i.e., in the fourth quadrant, it will be described by $\theta' = 3\pi/2 + \theta$.

TANDEM METHOD

(a) Normal Defect ($\alpha = \frac{\pi}{2}$)

(i) Contact measurement (Fig. 2)

The probe angle θ, or the standard probe angle θ' is known in this case.

From Fig. 2,

$$p_1 + p_2 - 2x = 2d \cot \theta' \tag{1}$$

x is thus given by:

$$x = \frac{1}{2}(p_1 + p_2) - d \cot \theta' \tag{2}$$

Also, $p_1 - x = (d - z) \cot \theta'$ (3)

which, combining with Eq (2) gives:

$$z = \frac{p_2 - p_1}{2 \cot \theta'} \tag{4}$$

It should be noted that the location of the defect does not depend upon knowing ℓ which is given by $\frac{-2d}{\sin \theta'}$.

(ii) Immersion measurement

The standard angle of incidence, θ_1', of the sound beam in water is known in this case.

p_1 and p_2 in Eq (1) - (4) will be replaced by:

$$p_1 = r_1 - (s_1 - d) \cot \theta_1' \tag{5}$$

$$p_2 = r_2 - (s_2 - d) \cot \theta_1' \tag{6}$$

and θ' is determined by the Snell's law generalized to all quadrants

$$\frac{\cos \theta'}{\cos \theta_1'} = \frac{v_{material}}{v_{water}} \tag{7}$$

where $v_{material}$ and v_{water} are velocities of sound waves propagating in the material and water respectively.

(b) Inclined Defect

(i) Contact measurement (Fig. 3)

Drawing the mirror image of the defect with respect to z = 0, it can be shown by trigonometry that:

$$p_1 - x = (d - z) \cot \theta' \tag{8}$$

$$p_2 - x = (d + z) \cot \bar{\phi} \tag{9}$$

$$\ell = -(d - z) \csc \theta' + (d + z) \csc \bar{\phi} \tag{10}$$

$$\text{where } \bar{\phi} = \theta' - 2\alpha \pm 2k\pi;\ k = 0, 1, 2, \text{---} \tag{11}$$

Eliminating x from Eq (8) and (9) gives:

$$z = \frac{p_2 - p_1 + d \cot \theta' - d \cot \bar{\phi}}{\cot \theta' + \cot \bar{\phi}} \tag{12}$$

Eliminating z from Eq (10) and (12), one gets:

$$(p_2 - p_1 + d \cot \theta' - d \cot \bar{\phi})(\csc \theta' + \csc \bar{\phi})$$

$$= (\ell + d \csc \theta' - d \csc \bar{\phi})(\cot \theta' + \cot \bar{\phi}) \tag{13}$$

$$\text{Let } a = \cot \bar{\phi} \tag{14a}$$

$$\text{then } \sqrt{1 + a^2} = \csc \bar{\phi} \tag{14b}$$

$$b = p_2 - p_1 + d \cot \theta' \tag{15}$$

$$c = \ell + d \csc \theta' \tag{16}$$

Substituting Eq (14) - (16) into Eq (13) and rearranging terms:

$$(b + d \cot \theta') \sqrt{1 + a^2} = c \cot \theta' - b \csc \theta' + a (c + d \csc \theta') \tag{17}$$

Squaring both sides of Eq (17) and solving the quadratic equation for a, the following equation is obtained:

$$a = \frac{-2(c \cot\theta' - b \csc\theta')(\cot \csc\theta') \pm \{4(c \cot\theta' - b \csc\theta')^2(c + d \csc\theta')^2 - 4[(\cot \csc\theta')^2 - (b + d \cot\theta')^2][(c \cot\theta' - b \csc\theta')^2 - (b + d \cot\theta')^2]\}^{1/2}}{2[(c + d \csc\theta')^2 - (b + d \cot\theta')^2]} \tag{18}$$

where b and c are given by Eq (15) and (16).

Equation (18) yields two roots of a, one of which is an extraneous root which arises from squaring Eq (17). The correct root can be determined by putting $\alpha = \pi/2$ and comparing the results with that given by section (a) for a normal defect. It is thus determined that the negative sign before the square root should be used. Knowing a, α can be found from Eq (14a) and (11).

$$\alpha = \frac{\theta' - \bar{\beta}}{2} \pm k\pi, \quad k = 0, 1, 2, \text{---} \tag{19}$$

Conventionally, α is considered to vary between 0 and π.

z and x can be determined from Eq (12) and (8) respectively.

(ii) Immersion measurement

p_1 and θ' in Eq (9) and (10) will be given by Eq (5) and (7) while p_2 is given by:

$$p_2 = r_2 - (s_2 - d) \cot \theta_2'$$

$$= r_2 - (s_2 - d) \frac{va}{(1 + a^2 - v^2a^2)^{1/2}} \tag{20}$$

$$\text{where} \quad v = \frac{v_{\text{water}}}{v_{\text{material}}} \tag{21}$$

b in Eq (15) thus becomes:

$$b = r_2 - p_1 + d \cot \theta' - (s_2 - d) \frac{va}{(1 + a^2 - v^2a^2)^{1/2}}$$

$$= b' - (s_2 - d) \frac{va}{(1 + a^2 - v^2a^2)^{1/2}} \tag{22}$$

Equation (17) thus becomes:

$$(b' + d \cot\theta') \sqrt{1 + a^2} - (s_2 - d) \frac{va}{(1 + a^2 - v^2a^2)^{1/2}} (\csc \theta' + \sqrt{1 + a^2})$$

$$= c \cot \theta' - b' \csc \theta' + a (c + d \csc \theta') \tag{23}$$

where c is given by Eq (16)

$$c = \ell + d \csc \theta'$$

The travel time t is given by:

$$t = \frac{\ell_1}{v_{water}} + \frac{\ell}{v_{material}} + \frac{\ell_2}{v_{water}} \tag{24}$$

where $\ell_1 = -(s_1 - d) \csc \theta_1'$ (25)

$$\ell_2 = -(s_2 - d) \csc \theta_2'$$

$$= (s_2 - d) \left(\frac{1 + a^2}{1 + a^2 - v^2a^2} \right)^{1/2} \tag{26}$$

ℓ is given by Eq (24) - (26)

a can be obtained from Eq (23) by numerical analysis. Alternatively, it can be arranged that the receiver transducer scans as close to the surface of the material as possible, such that $s_2 - d \to 0$. In this case, a, will be given by Eq (18) with b' replacing b.

DIFFERENT CASES OF INCLINED DEFECT

For simplicity, only the contact measurement is considered. This can be easily generalized to the immersion measurement. The angle of orientation, α, of the defect can range between 0 and π which affects the direction of the reflected echo from the defect. Ten different orientations are shown in Fig. 4.

Case (b) occurs when $2\alpha - \theta' + 2\pi = \pi$, i.e., $\alpha = 1/2 (\theta' - \pi)$. As $p_2 \to -\infty$, the defect cannot be detected. A similar situation occurs in case (h) when $2\alpha - \theta' = 0$, i.e., $\alpha = \theta'/2$ and $p_2 \to \infty$.

The double transducer oblique beam method has to be used in cases (a) and (i). This method will be described in the next section. The tandem method applies for cases (c), (e), (f) and (g). The receiver has to be scanned on the left hand side of the transmitter in cases (c) and (e) and on the right hand side in case (g). The transducer will be performed as transmitter/receiver in case (f).

Case (d) occurs when:

$$\theta' - 2\alpha = \frac{\pi}{2} \qquad \text{i.e.,} \quad \alpha = \frac{1}{2}\left(\theta' - \frac{\pi}{2}\right)$$

$$\text{As} \quad \ell = 2\,[-(d - z)\csc\theta' + z] \tag{27}$$

$$z = \frac{\frac{l}{2} + d\csc\theta'}{1 + \csc\theta'} \tag{28}$$

and x is given by Eq (8).

Case (j) occurs when $\alpha = \theta' - \pi/2$. The single transducer oblique beam method should be used. x and z are given by:

$$x = p_1 + \frac{\ell}{2}\cos\theta' \tag{29}$$

$$z = d + \frac{\ell}{2}\sin\theta' \tag{30}$$

If the defect is detected after a half-skip, $\alpha = \theta'/2 - \pi$. x and z are given by:

$$x = p_1 + \frac{\ell}{2}\cos\theta' \tag{31}$$

$$z = -d - \frac{\ell}{2}\sin\theta' \tag{32}$$

DOUBLE TRANSDUCER OBLIQUE BEAM METHOD

(i) Contact measurement (Fig. 5)

From Fig. 5, trigonometry shows:

$$p_1 - x = (d - z) \cot \theta' \tag{33}$$

$$p_2 - x = (d - z) \cot \phi \tag{34}$$

$$\ell = -(d - z) \csc \theta' + (d - z) \csc \phi \tag{35}$$

$$\text{where } \phi = 2\alpha - \theta' \pm 2k\pi \qquad k = 0, 1, 2,--- \tag{36}$$

Eliminating x from Eq (33) and (34) gives:

$$z = d - \frac{p_2 - p_1}{\cot \phi - \cot \theta'} \tag{37}$$

Eliminating z from Eq (35) and (37), one gets:

$$(p_2 - p_1)(-\csc \theta' + \csc \phi) = \ell(\cot \phi - \cot \theta') \tag{38}$$

$$\text{Let } g = \cot \phi \tag{39a}$$

$$\text{then } \sqrt{1 + g^2} = \csc \phi \tag{39b}$$

$$f = p_2 - p_1 \tag{40}$$

Substituting Eq (39) and (40) into Eq (38) and rearranging terms

$$f\sqrt{1 + g^2} = \ell g - \ell \cot \theta' + f \csc \theta' \tag{41}$$

Squaring both sides of Eq (41) and solving the quadratic equation for g

$$g = \frac{\ell(\ell \cot \theta' - f \csc \theta') \pm f[\ell^2 + (\ell \cot \theta' - f \csc \theta')^2 - f^2]^{1/2}}{\ell^2 - f^2} \tag{42}$$

One of the two roots is extraneous. Taking a special case, e.g. $\theta' = 5\pi/4$ and $p_1 - x = p_2 - p_1 = 1$, it can be determined that the positive sign should be taken. α can be determined from (39a) and (36).

$$\alpha = \frac{\theta' + \phi}{2} \pm k\pi, \quad k = 0, 1, 2,--- \tag{43}$$

z and x can be determined from Eq (37) and (33) respectively.

(ii) Immersion method

p_1 and θ' in Eq (33) - (35) will be given by Eq (5) and (7) while p_2 is given by:

$$p_2 = r_2 - (s_2 - d)\left\{\frac{vg}{(1 + g^2 - v^2g^2)^{1/2}}\right\} \tag{44}$$

f in Eq (40) thus becomes:

$$\begin{aligned} f &= r_2 - p_1 - (s_2 - d)\left\{\frac{vg}{(1 + g^2 - v^2g^2)^{1/2}}\right\} \\ &= f' - (s_2 - d)\left\{\frac{vg}{(1 + g^2 - v^2g^2)^{1/2}}\right\} \end{aligned} \tag{45}$$

Equation (41) becomes:

$$\begin{aligned} &f'\sqrt{1 + g^2} - (s_2 - d)\ \frac{vg}{(1 + g^2 - v^2g^2)^{1/2}}\ (-\csc\theta' + \sqrt{1 + g^2}) \\ &\quad = \mathfrak{t}g - \mathfrak{t}\cot\theta' + f'\csc\theta' \end{aligned} \tag{46}$$

where $\mathfrak{t}$ is given by Eq (24) - (26) with a replaced by g.

g can be solved from Eq (46) by numerical analysis. Alternatively, if the receiver transducer is scanned as close to the surface of the material as possible, i.e., $s_2 - d \to 0$, g can be determined by Eq (42) with f' replacing f.

COMPARISON BETWEEN THE TANDEM METHOD AND THE DOUBLE TRANSDUCER OBLIQUE BEAM METHOD

Equation (8) - (10) of the tandem method look similar to Eq (33) - (35) of the double transducer oblique beam method. The formulation of the beam path incident on the defect is the same in both cases. In the case of the tandem method, for sound wave reflected from the defect to the receiver transducer, the reflected beam can be considered to come from the mirror image of the defect with respect to the z = 0 plane. A sound beam having a direction of $-\gamma$ with the positive x axis will change its direction to γ after reflection from the z = 0 plane. Thus for the reflected beam in the double transducer oblique beam method, replacing z by -z and ϕ by $-\phi = \bar{\phi}$ will yield the formulation of the tandem method.

While the normal beam method can be considered as a particular case of the single transducer oblique beam method which in turn can be considered as a particular case of the double transducer oblique beam method when the reflected beam coincides with the incident beam, the double transducer oblique beam method may be considered as a particular case of the tandem method when no reflection occurs other than at the defect.

DISCUSSION

(i) Two transducers (transmitter and receiver)

When a defect is detected the question of determining whether the tandem methods applies or the double transducer oblique beam method applies arises. If $\ell \leq 2d$, then it is obvious that the latter should have been used. For $\ell > 2d$, if the defect is detected by the former method and the latter method is used to analyse the result, there are cases which result in $z < 0$, indicating that the former method should be used to analyze the result. If the defect is detected by the latter method and the former method is used to analyze the result, there are cases which result in a complex number for a, indicating that the latter method should have been used. But there are cases which yield solutions for both methods. In these situations another probe angle θ, needs to be used to resolve the ambiguity.

(ii) One transducer (transmitter/receiver)

It can be seen from Fig. 4 that one transducer can sometimes be used to detect a defect in cases (d) (f) and (j). If $\ell \leq 2d$, the single transducer oblique beam method, i.e., case (j) applies. If $\ell > 2d$, the situation cannot be resolved unless another probe angle is also used.

CONCLUSION

Both tandem method and double transducer oblique beam method are discussed with regard to locating inclined defects. For defect mapping in automatic ultrasonic testing, both methods should be used to analyse the data. When ambiguity in the location of defects arises, another transmitting transducer with a different angle of incidence should be able to resolve the difficulty.

REFERENCES

1. Udagawa, T., Uneno, T. and Isono, E. "Automatic ultrasonic testing and recording system for welds utilizing a microcomputer"; Materials Evaluation 40 (March 1982) pp 305-311.

2. Seiger, H. "Comparison of three flaw-location methods for automated ultrasonic testing"; NDT International 15 (June 1982) pp 131-135.

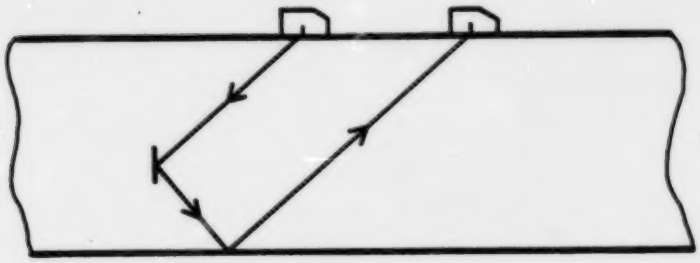

Fig. 1. Tandem method. (neg 10092)

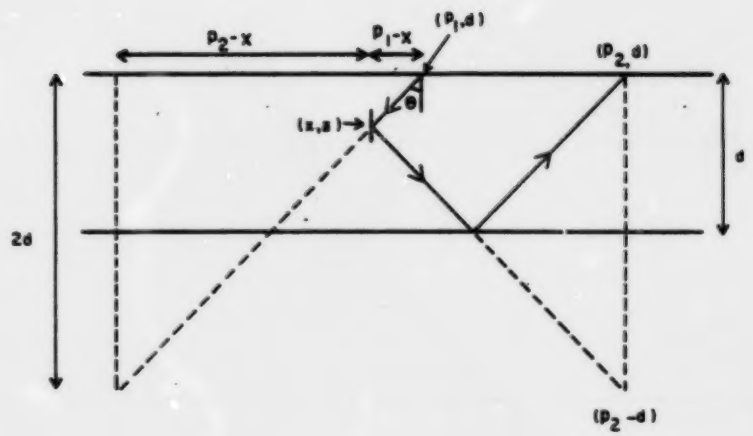

Fig. 2. Location of normal defect using the tandem method. (neg 10086)

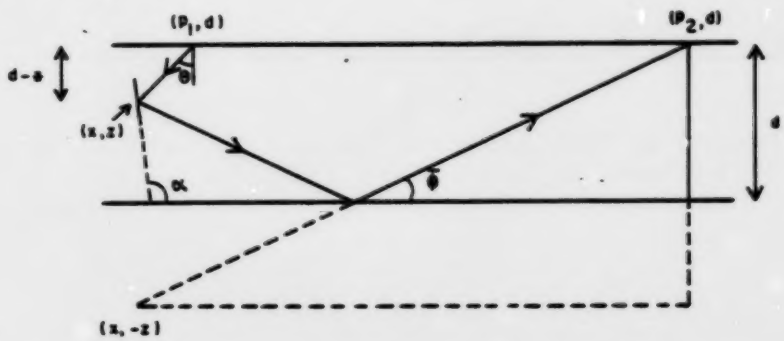

Fig. 3. Location of inclined defect using the tandem method. (neg 10090)

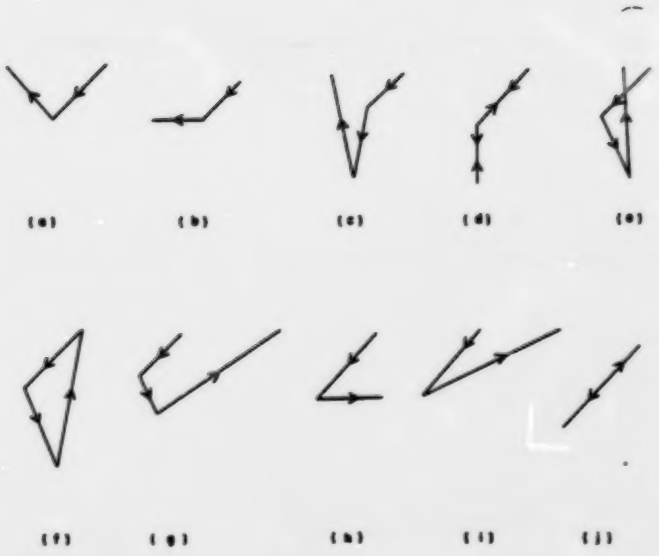

Fig. 4. Defects inclined at different orientations. (neg 10093)

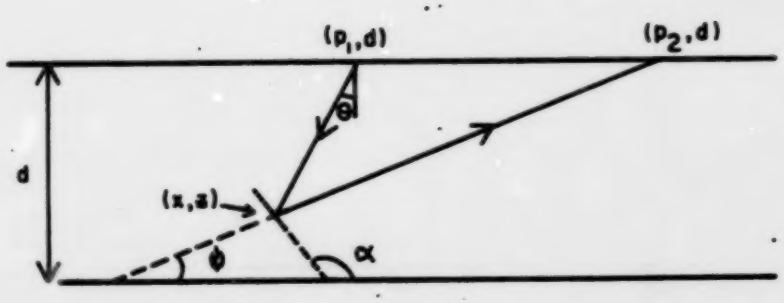

Fig. 5. Location of inclined defect using the double transducer oblique beam method. (neg 10087)